

Verification of Ohm's Law, Kirchoff's Voltage Law and Kirchoff's Current Law

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Abstract

The purpose of this report is to verify Ohm's law, Kirchoff's Current Law and Kirchoff's Voltage Law. Ohm's law relates voltage to resistance and current; Kirchoff's laws deal solely with current and voltage. A circuit was built using a given schematic. This circuit was also drawn in P-Spice. The circuit was then analyzed using three separate methods: the three laws, the P-Spice simulation and a digital multi-meter. Ohm's law and Kirchoff's laws were found to be valid after comparing the results from all three tests.

1.0 Introduction

Ohm's law, Kirchoff's Voltage Law and Kirchoff's Current Law are essential in the analysis of linear circuitry. Kirchoff's laws deal with the voltage and current in the circuit. Ohm's law relates voltage, current and resistance to one another. These three laws apply to resistive circuits where the only elements are voltage and/or current sources and resistors. Using the three laws any resistance of, current through or voltage across a resistor can be found if any two are already known. The purpose of this report is to provide verification of these laws.

Section 2 outlines the three laws and gives simple examples and conventions. Section 3 analyzes the circuit based solely on the schematic and generates the three equations needed for further analysis. Section 4 contains a circuit simulation run in P-Spice. Section 5 contains the data and calculations from the measurement of the physical circuit. Section 5 also contains the error analysis between the methods. Section 6 is the results discussion section. Section 7 is the experimental results and conclusions section.

2.0 Circuit Analysis Techniques

The following sections describe the three basic laws that are used in analyzing linear circuits.

2.1 Ohm's Law

Ohm's law is used to relate voltage to current and resistance. It states that voltage is directly proportional to current and resistance. This is stated mathematically as

$$V = IR, \quad (1)$$

where V is the voltage across an element of the circuit in volts, I is the current passing through the element in amps and R is the resistance of the element in ohms. Given any two of these quantities Ohm's law can be used to solve for the third.

2.2 Kirchoff's Voltage Law

Kirchoff's Voltage Law (KVL) states that the sum of all voltages in a closed loop must be zero. A closed loop is a path in a circuit that doesn't contain any other closed loops. Loops 1 and 2 in Figure 1 are examples of closed loops.

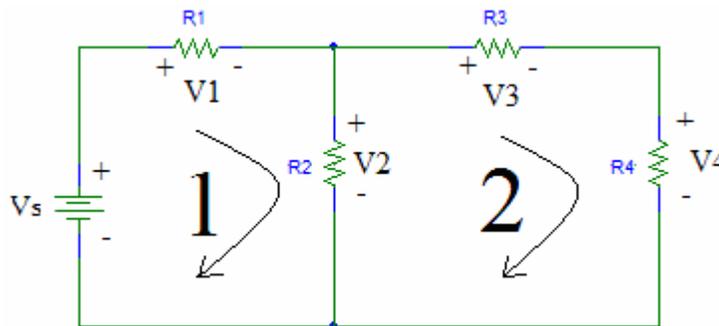


Figure 1: An example of KVL

The perimeter of the circuit is also a closed loop, but since it includes loops 1 and 2 it would be repetitive to include a KVL equation for it. If loop 1 is followed clockwise the KVL equation is

$$V_1 + V_2 - V_s = 0. \quad (1)$$

This equation holds true only if the passive sign convention is satisfied. In the case of KVL the passive sign convention states that when a positive node is encountered while following a loop the voltage across the element is positive. If a negative node is encountered the corresponding element voltage is negative. In order to simplify the KVL equations, the polarities should be assigned to satisfy the passive sign convention whenever possible.

2.3 Kirchoff's Current Law

Kirchoff's Current Law (KCL) deals with the currents flowing into and out of a given node. KCL states that the sum of all currents at a node must equal zero. This is illustrated in Figure 2.

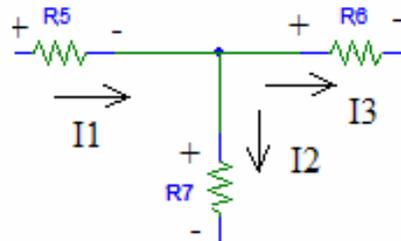


Figure 2: An example of KCL

The equation obtained by KCL for the node shown in Fig. 2 is

$$I_1 - I_2 - I_3 = 0. \quad (2)$$

In the case of KCL the passive sign convention deals with the direction of currents with respect to the node. Currents entering the node must have opposite signs as those exiting the node. The passive sign convention with respect to KVL can also be applied to KCL. On many schematics the polarities of resistors are already assigned, so the directions of the currents should be assigned such that the current is entering the positive terminal. This will simplify later calculations.

3.0 Analysis

The circuit analyzed in this laboratory is shown in Figure 3.

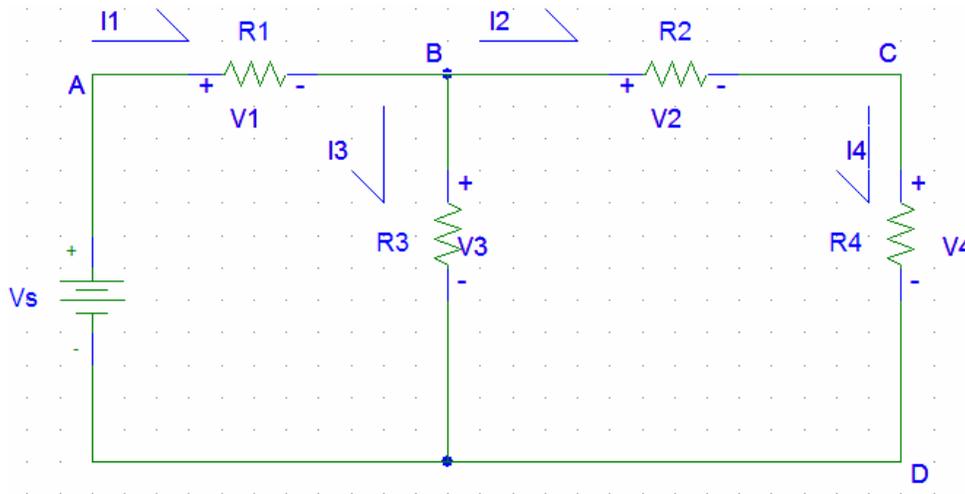


Figure 3: Resistive Circuit

The currents, voltages and polarities were labeled as shown on the given schematic. The current directions and voltage polarities have all been assigned such that the passive sign convention has been satisfied wherever possible.

The four resistors were chosen at random. Their resistance was then measured with a multi-meter. The nominal and measured resistance values are given in Table 1.

Table 1: Nominal and Measured Resistance Values

Resistor	Nominal Value ($k\Omega$)	Measured Value ($k\Omega$)
R_1	6.8	6.75
R_2	2.2	2.193
R_3	10	9.85
R_4	1	.994

KVL was applied to the two closed loops of the circuit using the symbolic labeling in Figure 3. The KVL equations are shown in (3) and (4).

$$\begin{aligned}
 & \text{KVL } A-B-D-A \\
 & V_1 + V_2 - V_s = 0 \qquad (3)
 \end{aligned}$$

$$\begin{aligned}
 & \text{KVL } B-C-D-B \\
 & V_2 + V_4 - V_3 = 0 \qquad (4)
 \end{aligned}$$

Current I_2 is the same as I_4 , so only node B generates a KCL equation. This equation is given in (5).

$$I_1 - I_2 - I_3 = 0 \quad (5)$$

This is the extent of the analysis that could be performed prior to measuring the voltages and currents. Before measurements were taken the circuit was simulated in P-Spice.

4.0 Simulation

The circuit schematic was entered into P-Spice. The resulting simulation schematic is given in Figure 4.

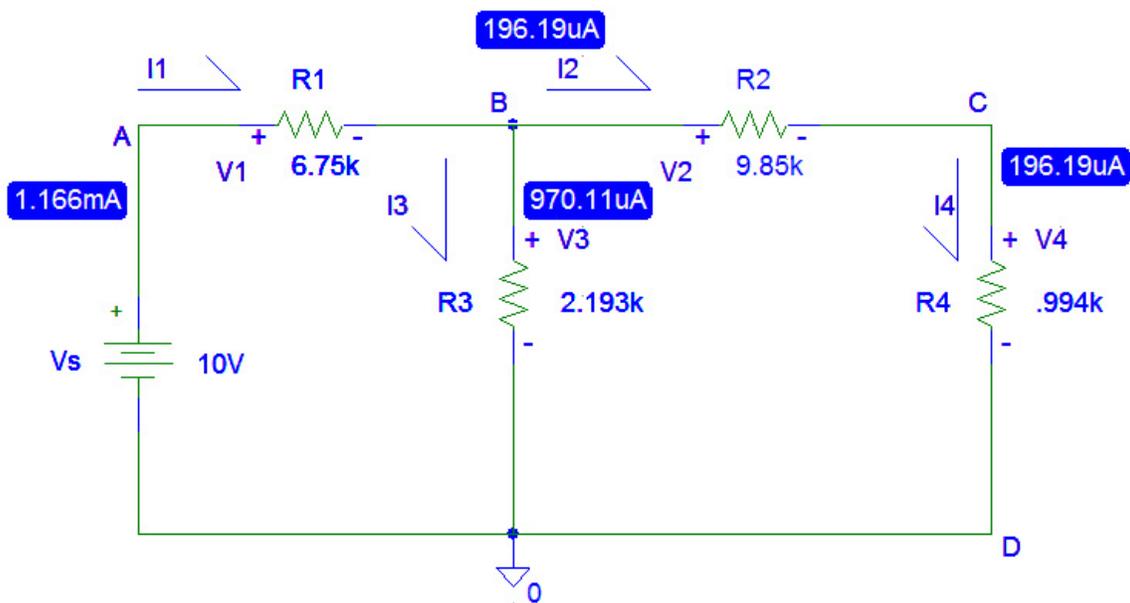


Figure 4: P-Spice simulation diagram with results

The measured resistance values were used in the simulation in order to obtain accurate results. The ground was arbitrarily placed to give P-Spice a reference node. The currents were then input into (5) to verify the equation.

$$1166.3 - 196.11 - 970.11 = 0 \quad (6)$$

Given that all three terms in (6) are in micro-amps, the P-Spice simulation does verify KCL. In order to use the KVL equations, Ohm's law had to be applied to the resistors and the current flowing through them. This gave the voltage across each resistor. These voltages were then inserted into (3) and (4).

$$7.8750 + 2.12745 - 10 = -.00205 \quad (7)$$

$$1.93247 + 0.195013 - 2.12745 = .000033 \quad (8)$$

The sum of the voltages in a closed loop should be zero if KVL is true. Neither (7) nor (8) produced zero volts. The sum of voltages produced while not exactly zero, were extremely close to zero. This was strong evidence that KVL is true.

5.0 Experimental Results

The circuit was constructed on a breadboard using the four random resistors. Ten volts was measured on the C.A.D.E.T. trainer's power supply and applied to the circuit. The voltages across and currents through each of the resistors was then measured and recorded in Table 2.

Table 2: Measured values of resistance and current

Voltage	Measured (V)	Current	Measured (mA)
V ₁	7.37	I ₁	1.08
V ₂	1.803	I ₂	0.81
V ₃	2.621	I ₃	0.25
V ₄	0.871		

The measured voltage values were inserted into (3) and (4) to verify KVL.

$$7.37 + 1.803 - 10 = -.827 \quad (9)$$

$$1.803 + 0.871 - 2.621 = 0.053 \quad (10)$$

The measured currents were inserted into (5) to verify KCL at node B.

$$1.06 - 0.81 - 0.25 = 0 \quad (11)$$

There was no error calculation performed on the KVL and KCL results because the expected value was zero. Ohm's law was used to calculate the resistance values using the values of the current through and voltage across each. A percent error analysis was then performed between the measured values and the Ohm's law values. A sample calculation from the analysis is shown in (12).

$$\%error = \left| \frac{measured - calculated}{measured} \right| \times 100\% = \left| \frac{6.75 - 6.82}{6.82} \right| \times 100\% = 1.03\% \quad (12)$$

These errors would ultimately be used to determine the validity of Ohm's law. The calculated resistances as well as the errors are given in Table 3.

Table 3: Measured, calculated and percent error for resistances

Resistor	Measured Value ($k\Omega$)	Calculated Value ($k\Omega$)	% Error
R ₁	6.75	6.82	1.03%
R ₂	2.193	2.23	1.66%
R ₃	9.85	10.5	6.19%
R ₄	.994	1.08	7.96%

6.0 Discussion

The simulation of the circuit verified KCL. This was shown by (6). P-Spice found the sum of the two currents exiting the node to be equal to the current entering the node. The experimental results of the KCL test were not so close. There was less than two percent error between the measured and calculated I_1 's. This error is acceptable in stating that KCL is true.

The simulated KVL test was not as expected. Neither (7) nor (8) produced zero volts. However both of the equations added up to voltages small enough to be negligible in stating that KVL is valid. As with the KCL tests, the experimental KVL results in (9) and (10) did not meet the predicted values. Both of the equations produced values within

one volt away from zero. This was a larger discrepancy than that found in (7) and (8), but it was still small enough to conclude that KVL is valid.

All of the percent errors for the calculated versus measured resistance were less than eight percent. The errors appeared to increase as the resistor number increased (i.e. R_1 , R_2 etc.). This was probably due to error accumulation in the calculations. Slight errors such as rounding in intermediate calculation steps would account for the increase. Another possible source of error was in the values of the resistors chosen. The resistors all had values in the kilo-ohm range. Such large resistance values would make it difficult to accurately measure the small currents passing through them. This would account for part of the discrepancy. The errors were all fairly small which lead to the conclusion that Ohm's law is indeed valid.

A major source of error that applies to all three cases was the measuring process. If fingers were in contact with both leads of the multi-meter when taking resistance measurements the readings would be slightly off. This is because the meter would try to add the resistance of the body in the loop. The same type of problem could have occurred with the voltage and current measurements. If the connections were not fully made then the meter would have made inaccurate readings.

7.0 Conclusion

Ohm's law, KVL and KCL are three of the most basic techniques for the analysis of linear circuits. The purpose of this lab was to prove these three laws valid. A circuit was provided with four unknown voltages and three unknown currents. This circuit was constructed in P-Spice and simulated. Then the circuit was built on a breadboard. The voltage and current values were measured and placed into KVL and KCL equations to determine whether they turned out as predicted. These measured values were then used in the Ohm's law equation to find resistance. The calculated and measured values were then compared to the expected results from the theories in section 2.0. All of the discrepancies between tested and expected values were small, therefore all three of the laws could be considered valid.